

Make the math tangible.

Four short investigations connecting objects, mathematical expressions and a four-register RPN calculator.

STACKCALC / FRACTION STUDIO

Build the amount. Name the parts. Type the two dots.

WHOLE NUMERATOR DENOMINATOR RESULT X

WHOLE . NUM . DEN

Equal bases. Stack heights to compare one whole.

STACKCALC / STACK STAGE

Read from T to X. Move values after each action.

T fourth

Z third

Y first operand

X screen / result

ENTER copies X; next entry replaces X.

After arithmetic, a new number lifts the stack.

STACKCALC / EXPRESSION TREE

$(a + b) * (c - d)$ / visit left, right, then operator

Read out: $a + b + c - d - *$

STACKCALC / COORDINATE LAB

10 x 10 points / coordinates 0-9 / pitch 12 mm

A(1,2) to B(4,6): run 3, rise 4, distance 5

1 ENTER 4 - x^2 2 ENTER 6 - x^2

+ sqrt / function names, not literal key counts

BUILD. PREDICT. PRESS. EXPLAIN.

Use with StackCalc in four-register RPN mode. Suggested lessons: 15-25 minutes each, in pairs. Review copy: classroom timing and physical fit still need a pilot.

Where did the value go?

Goal: distinguish ENTER, number entry and arithmetic. Materials: stack board, four writable value tiles, calculator.

Start with 8 ENTER 3 -

Begin with all four registers at zero. One learner presses keys; a partner updates the tiles. Read T, Z, Y, X from top to bottom. Include the zeros.

After action	T	Z	Y	X
8	0	0	0	8
ENTER	0	0	8	8
3	0	0	8	3
-	0	0	0	5

Now type 2, then press *.

Predict before pressing: the result 5 lifts to Y as 2 enters X. Multiplication leaves 10 in X. ENTER was unnecessary between the subtraction and the new number.

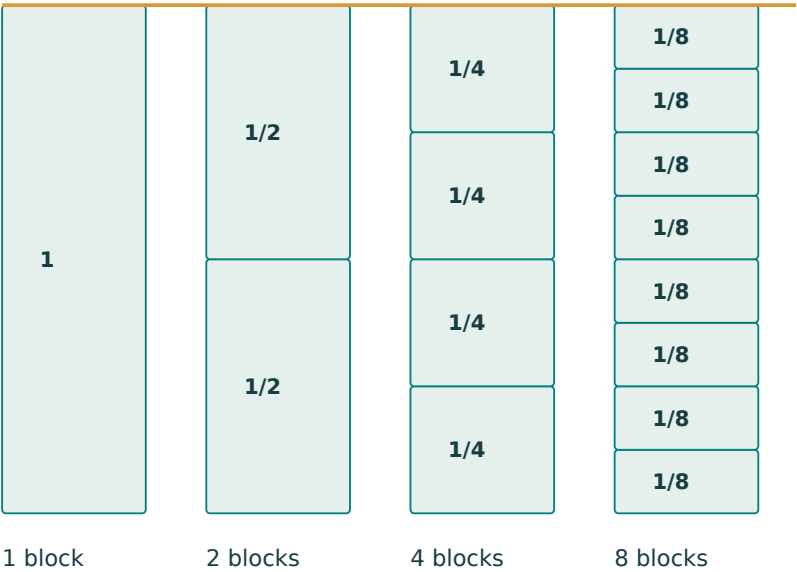
Say it precisely

ENTER copies X up and disables the next entry lift. A binary operator combines Y and X; the answer appears in X.

Teacher prompt: What would happen with 3 ENTER 8 -? Explain why subtraction and division must keep operand order.

Build it. Name it. Enter it.

Goal: connect proportional height, fraction notation and dot entry. Materials: fraction tray, tower blocks, numeral tiles, calculator.



Try 3/4

Stack three quarter-blocks beside one whole. Compare from the same floor. Name the numerator and denominator.

- 1 Represent

Put 0, 3 and 4 into the whole, numerator and denominator wells. Leave the result well free.
- 2 Enter

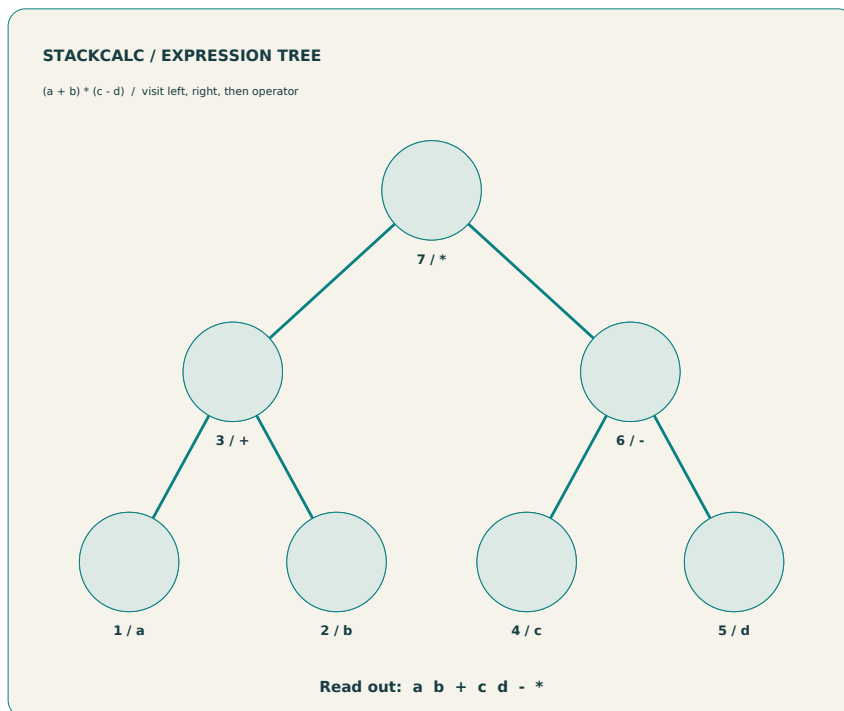
Type . 3 . 4, then ENTER. Show 2 1/2 as two whole towers and one half tower. Type 2 . 1 . 2, then +.
- 3 Explain

Regroup: $3/4 + 2\ 1/2 = 3\ 1/4$. The result may display 3.25 or a mixed fraction, depending on display mode.

Transfer: show why $1/2 = 2/4 = 4/8$. Equal stacks reach the same height. Typing a dot does not perform division.

Read the structure.

Goal: turn a grouped expression into an evaluation order. Materials: tree board and seven tokens, value tiles, calculator.



Left

Right

Operator

Point to each well in numbered order. Speak the token after both its children.

(3 + 2) * (8 - 4)

Set a=3, b=2, c=8, d=4. Read: 3 2 + 8 4 - *. On the calculator use: 3 ENTER 2 + 8 ENTER 4 - *. Result: 20.

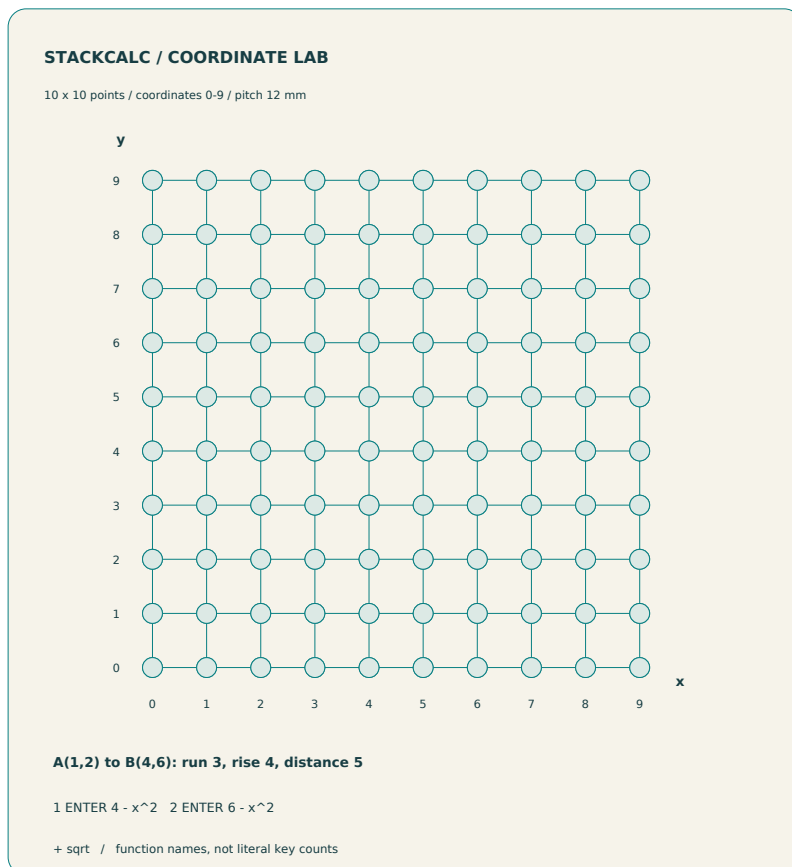
Postfix tokens are not a literal key list

ENTER separates consecutive number entries. After +, typing 8 automatically lifts the subtotal 5. Preserve the left/right operand order.

Transfer: compare $3 * 8 - 4 + 1$ with $3 * (8 - (4 + 1))$. Sketch their different trees before calculating.

Measure the diagonal.

Goal: connect run, rise and the Pythagorean theorem. Materials: coordinate board, three pegs, loose yarn, calculator.



$$A = (1, 2)$$

$$B = (4, 6)$$

Place the third peg at (4, 2). Trace the horizontal, vertical and diagonal paths using loose yarn.

Count intervals: run 3, rise 4.
Each grid interval represents one coordinate unit.

$$1 \text{ ENTER } 4 - x^2$$

$$X = 9$$

$$2 \text{ ENTER } 6 - x^2$$

$$X = 16, Y = 9$$

$$+ \text{ sqrt}$$

$$X = 5$$

Discuss: the differences are negative, but their squares are positive. The distance is 5 coordinate units, not 5 mm. Count spaces between pegs.

Predict before you press.

Name: _____ Partner: _____

01 / STACK

Record T, Z, Y, X after 3 ENTER 8 ENTER 4 ENTER 1. Then predict +, -, *.

02 / FRACTIONS

Draw stacked blocks showing $\frac{1}{2} + \frac{1}{4}$. Write the two-dot keystrokes and the result.

03 / TREE

Use $(6 + 1) * (9 - 5)$. Label the tree, write postfix tokens, then calculator actions.

04 / DISTANCE

Find the distance from (2, 1) to (5, 5). Explain which two lengths you squared.

Listen for the reasoning.

Accept equivalent notation. Ask learners to explain the representation, the operation and the result.

01 / Stack

Before +: T=3, Z=8, Y=4, X=1. After +: T=3, Z=3, Y=8, X=5. After -: T=3, Z=3, Y=3, X=3. After *: T=3, Z=3, Y=3, X=9. The top register repeats on a binary stack drop.

02 / Fractions

One half plus one quarter equals three quarters. Keys: .1.2 ENTER .1.4 +. A decimal result of 0.75 is equivalent. Ask why the denominators are not simply added.

03 / Tree

Postfix: 6 1 + 9 5 - *. Calculator: 6 ENTER 1 + 9 ENTER 5 - *. Subtotals 7 and 4 give 28. A tree describes dependencies; it does not prescribe an ENTER for every token.

04 / Distance

Run 3, rise 4, distance 5. Keys: 2 ENTER 5 - x^2 1 ENTER 5 - x^2 + sqrt. Square each difference before adding; take one square root at the end.

Two different examples

$3 * 8 - 4 + 1 = 21$; $3 * (8 - (4 + 1)) = 9$. Grouping changes the tree and the value, not just the button rhythm.

Make a classroom set.

One of each board per pair. These are design prototypes for trial lessons, not production-approved retail products.

Print the paper kit

Choose Letter or A4. Print actual size and measure the 50 mm scale on the card sheet. Cards are single-sided. The booklet is in reading order; duplex long-edge works for a stapled handout. Saddle-stitch imposition and commercial bleed are not supplied.

Make the physical kit

Start with the fit coupon and a peg. Fraction blocks share 24 x 24 mm bases; height is 96/n mm. Print flat contact faces and laser the labels afterward. Use separate focus-height jobs. Keep towers at or below one whole (96 mm); show mixed numbers in adjacent towers.

Prepare the lesson

Use writable film on blank tiles or paper liners. Keep loose pieces inventoried; this set is for supervised middle-school use. Do not assume PLA is dry-erase or tension the prototype pegs with bands. Use four-register RPN mode.

Release gate

Before selling printed kits: inspect edges, test repeated fitting and legibility, pilot the lessons with an educator, verify the target calculator's actions, and approve packaging and print proofs. Prices, licensing and shop listings remain undecided.

Reference and verification

HP 32SII Owner's Manual, chapters 1-2 and appendix B. Arithmetic examples are independently checked; this guide does not certify every StackCalc mode or key layout.