UNIT 12 TEACHER CORE LESSON • AP COMPUTER SCIENCE PRINCIPLES (GRADES 9–12 / AP CS • SC-32)

Positional Number Systems: Decimal, Binary & Hexadecimal

Data Representation & Binary Number Systems • Standard: CCSS.MATH.PRACTICE.MP5

1. MATHEMATICAL CONCEPT & STACK APPROACH: Decompose integers into powers of 2 and powers of 16 using [0] and [1] blocks on the Place Value Tray, and verify base conversions.

Binary (base-2) and hexadecimal (base-16) follow the exact same positional place-value principle as base-10, using powers of 2 (8, 4, 2, 1) or 16 instead of powers of 10. Every 4-bit binary group (0000 to 1111) maps to exactly one hexadecimal digit (0 to F).

2. MODELING WITH BLOCKS & STUDENT PREDICTION

Published Block Kit: Place Value Studio (Binary [0]/[1] Register Trays & Base-N Columns)

Hands-On Setup: Use the sockets of the Place Value Tray as binary bit positions (8, 4, 2, 1). Place [1] Digit Blocks for active '1' bits and [0] Digit Blocks for '0' bits.

Student Prediction (Before Calculator): Place [1], [0], [1], [1] in the 8, 4, 2, 1 sockets: binary $1011_2 = 8 + 0 + 2 + 1 = 11_{10}$. In hexadecimal ($A$=10, B=11), $11_{10} = 0B_{16}$.

3. CALCULATOR EXECUTION (SC-32 KEYCAPS & 4-LEVEL STACK TRACE)

Keystroke Flow: [1] [1] [↶] [BASE]

Key Pressed	Screen (X)	Internal 4-Level Stack Effect (T, Z, Y, X)
[1] [1]	`11`	Enter decimal integer 11 in DEC mode
[↶] [BASE]	`HEX 0B`	Press [f:] [BASE]: screen displays Hexadecimal 0B!
[↶] [BASE]	`BIN 1011`	Press [f:] [BASE]: screen displays Binary 1011!
[↶] [BASE]	`OCT 13`	Press [f:] [BASE]: screen displays Octal 13!
[↶] [BASE]	`DEC 11`	Press [f:] [BASE]: returns to Decimal 11

4. CONTEXT (Leibniz's Binary Medallion & Claude Shannon's Boolean Relay Circuits) • Thinking in the Stack (SC-32 Manual) — Ch. 1: 32,768 Hz Quartz Timebase + Ch. 10: Boolean Logic, Two's Complement & the 36-Bit Word ([BASE]) + Ch. 13: Coupled Systems

In 1697, Gottfried Wilhelm Leibniz designed a medallion showing that every integer can be built from 1 and 0 alone using powers of 2 (8, 4, 2, 1)—and in 1937, 21-year-old MIT student Claude Shannon proved that electrical switches wired in series (AND) and parallel (OR) execute Boolean logic directly! Using [0] and [1] blocks on the Place Value Tray, students see how place value powers modern computing.

Core Investigation (Student Warm-Up A):

Express decimal 255 as a sum of powers of 2 to write its 8-bit binary and 2-digit hexadecimal representations.

Teacher Answer Key: Binary: 11111111_2 (eight 1s); Hexadecimal: FF_{16} ($15 \times 16 + 15 = 255$).

Bitwise Logic: AND Masking & Column-by-Column Comparison

Boolean Logic & Bitwise Operations • Standard: CCSS.MATH.PRACTICE.MP5

1. FOLLOW-UP EXTENSION CONCEPT, BLOCKS & PREDICTION

Concept & Stack Approach: Bitwise logic operations (AND, OR, XOR) compare corresponding binary place-value columns of two integers in parallel. Placing value A in the top row and mask B in the bottom row lets you inspect each bit column directly.

Blocks & Prediction: Row 1 of the Place Value Tray holds nibble A ($[1][1][0][0]_2 = 12$). Row 0 holds mask B ($[1][0][1][0]_2 = 10$). Prediction: Bitwise AND: bit 3 (1 & 1 = 1), bit 2 (1 & 0 = 0), bit 1 (0 & 1 = 0), bit 0 (0 & 0 = 0). Result: $1000_2 = 8_{10}$.

2. EXTENSION CALCULATOR SEQUENCE & STACK TRACE

[1] [2] [ENTER] [1] [0] [AND]

Key Pressed	Screen (X)	Internal 4-Level Stack Effect
[1] [2]	12	Enter operand $A = 12$ (binary 1100)
[ENTER]	12	12 lifted to register Y
[1] [0]	10	Enter operand $B = 10$ (binary 1010) in X
[AND]	8	Bitwise AND evaluates 1100 & 1010 = 1000 (8 decimal)!

3. TARGET MISCONCEPTIONS & HANDS-ON BLOCK REMEDIATIONS

Misconception 1: Confusing bitwise [AND] / [OR] with decimal addition or multiplication (e.g., thinking 12 AND 10 is 22).

Hands-On Block Remediation: Model 12 ($[1][1][0][0]_2$) in Row 1 and 10 ($[1][0][1][0]_2$) in Row 0 of the Place Value Tray using [1] and [0] Digit Blocks; only columns where BOTH rows hold [1] remain 1 in 12 AND 10 = 8 ($[1][0][0][0]_2$).

Misconception 2: Reading hexadecimal [10] as ten instead of decimal 16 ($1 \times 16^1 + 0 \times 16^0$).

Hands-On Block Remediation: Place [1] in the Sixteens column and [0] in the Ones column on the Place Value Tray, then switch from [HEX] [10] to [DEC] on the SC-32 to see 16.

Extension Challenge (Student Warm-Up B):

Compare 12 and 3 column by column in 4-bit binary to find the result of 12 OR 3 in both binary and decimal.

My Paper Answer: _____ **[ ] Verified on SC-32 (Online Key: stackcalc.io/curriculum/answers)**

STUDENT LAB WORKSHEET • UNIT 12 (SC-32)

Digital Logic, Base Conversions & Computer Architecture

Name: _____ Date: _____ Class: _____

STUDENT REFERENCE GUIDE: POSITIONAL NUMBER BASES & BITWISE LOGIC RULES

- Binary (Base 2) uses powers of 2 (..., 8, 4, 2, 1) with digits 0 and 1; Hexadecimal (Base 16) groups 4 binary bits into a single digit 0–9 or A–F.
- Bitwise AND (&): output bit is 1 only when BOTH input bits in that column are 1 (used for masking and network subnets).
- Bitwise OR (|): output bit is 1 when EITHER input bit is 1. Bitwise XOR (^): output bit is 1 when the two input bits DIFFER.

Key Sequence: 1 1 ↰ BASE

Investigation 1: Decompose decimal 42 into powers of 2 to write its 6-bit binary representation.

1. Binary Place-Value / Bit Grid:



2. Base Conversion / Bitwise Steps:

Place Weights (8-4-2-1):

Bit Pattern / Logic Mask:

Decimal / Binary / Hex Result:

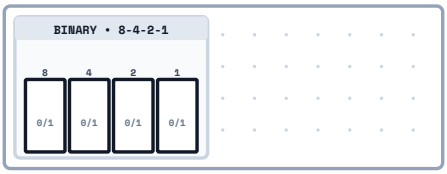
3. My Prediction:

Final Verified Answer:

[ ] Checked on SC-32

Investigation 2: Convert hexadecimal $2A_{16}$ to decimal using place value.

1. Binary Place-Value / Bit Grid:



2. Base Conversion / Bitwise Steps:

Place Weights (8-4-2-1):

Bit Pattern / Logic Mask:

Decimal / Binary / Hex Result:

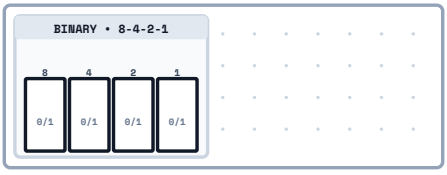
3. My Prediction:

Final Verified Answer:

[ ] Checked on SC-32

Investigation 3: Evaluate the bitwise AND of 15 and 7 in binary and decimal.

1. Binary Place-Value / Bit Grid:



2. Base Conversion / Bitwise Steps:

Place Weights (8-4-2-1):

Bit Pattern / Logic Mask:

Decimal / Binary / Hex Result:

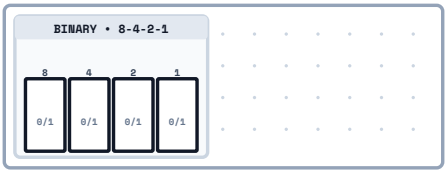
3. My Prediction:

Final Verified Answer:

[ ] Checked on SC-32

Investigation 4: Evaluate the bitwise XOR of 12 and 10 in binary and decimal.

1. Binary Place-Value / Bit Grid:



2. Base Conversion / Bitwise Steps:

Place Weights (8-4-2-1):

Bit Pattern / Logic Mask:

Decimal / Binary / Hex Result:

3. My Prediction:

Final Verified Answer:

[ ] Checked on SC-32

UNIT 12 PRACTICE WORKSHEET A • GRADES 9–12 / AP CS (SC-32)

Unit 12 Practice A: Core Fluency

Name: _____ Date: _____ Score: _____ / 10

TIER 1 • LEVEL 1: BUILDER FOUNDATION • STUDY THESE 2 WORKED EXAMPLES BEFORE SOLVING PRACTICE A (#1–#10) ON PAPER FIRST

EXAMPLE 1 Convert binary 1101_2 to decimal using 8-4-2-1 weights.

$1(8) + 1(4) + 0(2) + 1(1) = 8 + 4 + 1 = 13$.

Verified Answer: 13

EXAMPLE 2 Convert 8-bit binary 10101100_2 to Hexadecimal.

Split into two 4-bit nibbles: $1010_2 = 10 = A$, and $1100_2 = 12 = C \rightarrow \mathrm{AC}_{16}$.

Verified Answer: AC_{16}

1 Tier 1 • Builder

Convert 4-bit binary 0101_2 to decimal:

BINARY • 8-4-2-1

8	4	2	1
0/1	0/1	0/1	0/1

Paper Step: _____ | | SC-32 Ans: _____

2 Tier 1 • Builder

Convert 4-bit binary 1011_2 to decimal:

BINARY • 8-4-2-1

8	4	2	1
0/1	0/1	0/1	0/1

Paper Step: _____ | | SC-32 Ans: _____

3 Tier 1 • Builder

Convert 4-bit binary 1111_2 to decimal and hexadecimal:

BINARY • 8-4-2-1

8	4	2	1
0/1	0/1	0/1	0/1

Paper Step: _____ | | SC-32 Ans: _____

4 Tier 1 • Builder

Convert decimal 9 to 4-bit binary:

BINARY • 8-4-2-1

8	4	2	1
0/1	0/1	0/1	0/1

Paper Step: _____ | | SC-32 Ans: _____

5 Tier 1 • Builder

Convert decimal 14 to 4-bit binary and hexadecimal:

BINARY • 8-4-2-1

8	4	2	1
0/1	0/1	0/1	0/1

Paper Step: _____ | | SC-32 Ans: _____

6 Tier 1 • Builder

Convert hexadecimal $1A_{16}$ to decimal:

BINARY • 8-4-2-1

8	4	2	1
0/1	0/1	0/1	0/1

Paper Step: _____ | | SC-32 Ans: _____

7 Tier 1 • Builder

Convert hexadecimal $2F_{16}$ to decimal:

BINARY • 8-4-2-1

8	4	2	1
0/1	0/1	0/1	0/1

Paper Step: _____ | | SC-32 Ans: _____

8 Tier 1 • Builder

Convert hexadecimal FF_{16} to decimal:

BINARY • 8-4-2-1

8	4	2	1
0/1	0/1	0/1	0/1

Paper Step: _____ | | SC-32 Ans: _____

9 Tier 1 • Builder

Convert 8-bit binary 11000111_2 to hexadecimal:

BINARY • 8-4-2-1

8	4	2	1
0/1	0/1	0/1	0/1

Paper Step: _____ | | SC-32 Ans: _____

10 Tier 1 • Builder

Convert octal 25_8 to decimal:

BINARY • 8-4-2-1

8	4	2	1
0/1	0/1	0/1	0/1

Paper Step: _____ | | SC-32 Ans: _____