UNIT 11 TEACHER CORE LESSON • AP STATISTICS (GRADES 11–12 / AP • SC-32)

Combinations vs. Permutations: Subsets & Prime Cancellation

Probability, Random Variables & Combinatorics • Standard: CCSS.MATH.HSS.CP.B.9

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1. MATHEMATICAL CONCEPT & STACK APPROACH: Calculate permutations and combinations by canceling common factorial terms on paper and verifying on the stack.

Combinations nCr = $n!$ / (r!(n-r)!) count unordered selections by dividing ordered permutations by the r! duplicate arrangements of the chosen group. Canceling 5! from 8! leaves (8 × 7 × 6) / 3! = 56.

2. MODELING WITH BLOCKS & STUDENT PREDICTION

Block Kit & Mathematical Focus: Prime Factor Studio (Factorial Cancellation) & Slot Diagram

Hands-On Setup: Use Prime Factor blocks to model prime factor cancellation in 8! / (3! × 5!) = (8 × 7 × 6) / (3 × 2 × 1) = 8 × 7 = 56.

Student Prediction (Before Calculator): How many ways to choose a committee of 3 from 8 candidates? 8C3 = 8! / (3! × 5!) = (8 × 7 × 6) / 6 = 56.

3. CALCULATOR EXECUTION (SC-32 KEYCAPS & 4-LEVEL STACK TRACE)

Keystroke Flow: 8 ↶ x! 3 ↶ x! ÷ 5 ↶ x! ÷

Key Pressed	Screen (X)	Internal 4-Level Stack Effect (T, Z, Y, X)
8	8	Total candidates n = 8
↶ x!	40320	8! = 40,320 in register X
3	3	Subset r = 3 (40320 lifts to Y)
↶ x!	6	3! = 6 in X
÷	6720	40320 / 6 = 6720 in X
5	5	Remaining n - r = 5 (6720 lifts to Y)
↶ x!	120	5! = 120 in X
÷	56	6720 / 120 = 56 combinations!

4. CONTEXT (Sir Francis Galton's 1873 Bean Machine & The Mathematics of Small Samples) • Thinking in the Stack (SC-32 Manual) — Ch. 9: Least Squares, Small Samples & Uncertainty (Gauss's Ceres Recovery, Gosset, [PROB], [z^t], [nCr], [nPr] & [Σ+])

In 1873, Sir Francis Galton built a vertical board studded with pins (the Quincunx) to show how balls bouncing left or right in combinations nCr settle into a bell-shaped curve—while in 1908 in Dublin, chemist William Sealy Gosset ('Student') developed small-sample statistics with n − 1 degrees of freedom! Students cancel factorial primes with Prime Factor blocks and model exact probability ratios.

Core Investigation (Student Warm-Up A):

A lottery draws 4 winning numbers from a pool of 10 (order does not matter). How many winning combinations are possible?

Teacher Answer Key: (10 × 9 × 8 × 7) / (4 × 3 × 2 × 1) = 5040 / 24 = 210 combinations.

Sample Mean & Variance of Discrete Data Sets

One-Variable Data Analysis & Summations • Standard: CCSS.MATH.HSS.ID.A.2

1. FOLLOW-UP EXTENSION CONCEPT, PAPER STEPS & PREDICTION

Concept & Stack Approach: Statistical measures of center (Mean $\bar{x}$) and spread (Standard Deviation s) summarize an entire distribution of values. Finding the sum of observations (Σx) and dividing by the sample count n locates the balance point of the data set.

Mathematical Setup & Prediction: Plot the 5 sample values {4, 6, 7, 8, 10} on a number-line dot plot to see how distances above and below 7 balance. Prediction: Sum = 4 + 6 + 7 + 8 + 10 = 35. Sample size $n = 5$. Mean $\bar{x} = 35 / 5 = 7.0$.

2. EXTENSION CALCULATOR SEQUENCE & STACK TRACE

[4] [Σ+] [6] [Σ+] [7] [Σ+] [8] [Σ+] [1] [0] [Σ+]

Key Pressed	Screen (X)	Internal 4-Level Stack Effect
[4] [Σ+]	1	First score entered: sample size $n = 1$
[6] [Σ+]	2	Second score: $n = 2$
[7] [Σ+]	3	Third score: $n = 3$
[8] [Σ+]	4	Fourth score: $n = 4$
[1] [0] [Σ+]	5	Fifth score: $n = 5$ total data points

3. TARGET MISCONCEPTIONS & HANDS-ON BLOCK REMEDIATIONS

Misconception 1: Forgetting to clear the statistical registers ([CLΣ]) before entering a new dataset with [Σ+].

Hands-On Block Remediation: Notice that pressing [Σ+] displays the sample count n in Register X; if the first data point shows $n > 1$, press [CLΣ] first so prior data does not corrupt the mean.

Misconception 2: Dividing by r! and then multiplying by $(n - r)!$ when computing C(n, r) = $n!$ / (r!$(n - r)$!).

Hands-On Block Remediation: Compute n! into Y, compute r! and $(n - r)$! and multiply them together in X ([×]), then press [÷] once to divide n! by the combined denominator.

Extension Challenge (Student Warm-Up B):

Given the four data points {12, 15, 18, 21}, find the sample size n, the total sum of the values, and the sample mean $\bar{x}$.

My Paper Answer: _____ **[ ] Verified on SC-32 (Online Key: stackcalc.io/curriculum/answers)**

STUDENT LAB WORKSHEET • UNIT 11 (SC-32)

Combinatorics, Probability Ratios & Series Accumulation

Name: _____ Date: _____ Class: _____

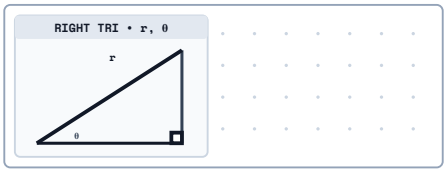
STUDENT REFERENCE GUIDE: COMBINATIONS, PERMUTATIONS & SAMPLE STATISTICS RULES

- Factorial n! = $n \times (n-1) \times \ldots \times 1$ counts the ways to arrange n distinct items; Prime Factor blocks show how common factors cancel in n! / r!.
- Permutations (order matters): nPr = $n!$ / $(n-r)!$ counts ordered arrangements of r items chosen from n.
- Combinations (order does not matter): nCr = $n!$ / (r! × $(n-r)!$) divides out the r! duplicate orderings.

Key Sequence: 8 ↶ x! 3 ↶ x! ÷ 5 ↶ x! ÷

Investigation 1: Evaluate 6! (the number of ways to arrange 6 distinct books on a shelf).

1. Slot Diagram / Data Sketch:



2. Factorial / Statistical Steps:

Formula (nPr, nCr, or Stat):

Factorial / Sum Expansion:

Computed Value:

3. My Prediction:

_____

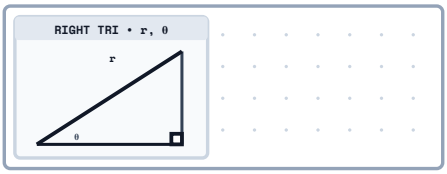
Final Verified Answer:

_____

[ ] Checked on SC-32

Investigation 2: How many ways can 5 runners finish in 1st, 2nd, and 3rd place?

1. Slot Diagram / Data Sketch:



2. Factorial / Statistical Steps:

Formula (nPr, nCr, or Stat):

Factorial / Sum Expansion:

Computed Value:

3. My Prediction:

_____

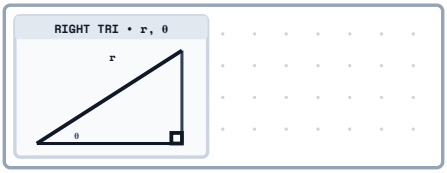
Final Verified Answer:

_____

[ ] Checked on SC-32

Investigation 3: How many 2-person partnerships can be formed from 7 students?

1. Slot Diagram / Data Sketch:



2. Factorial / Statistical Steps:

Formula (nPr, nCr, or Stat):

Factorial / Sum Expansion:

Computed Value:

3. My Prediction:

_____

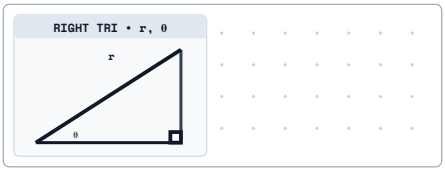
Final Verified Answer:

_____

[ ] Checked on SC-32

Investigation 4: Find the mean $\bar{x}$ of the two data values 5 and 15.

1. Slot Diagram / Data Sketch:



2. Factorial / Statistical Steps:

Formula (nPr, nCr, or Stat):

Factorial / Sum Expansion:

Computed Value:

3. My Prediction:

_____

Final Verified Answer:

_____

[ ] Checked on SC-32

UNIT 11 PRACTICE WORKSHEET A • GRADES 11–12 / AP (SC-32)

Unit 11 Practice A: Core Fluency

Name: _____ Date: _____ Score: _____ / 10

TIER 1 • LEVEL 1: BUILDER FOUNDATION • STUDY THESE 2 WORKED EXAMPLES BEFORE SOLVING PRACTICE A (#1–#10) ON PAPER FIRST

EXAMPLE 1 **Evaluate 6C2 (choosing 2 items from 6).**

6C2 = (6 × 5) ÷ (2 × 1) = 30 ÷ 2 = 15. Key: [6] [ENTER] [2] [nCr].

Verified Answer: 15

EXAMPLE 2 **How many ways can 4 runners finish 1st, 2nd, and 3rd (4P3)?**

Order matters: 4 × 3 × 2 = 24 ways.

Verified Answer: 24

1 Tier 1 • Builder

Evaluate factorial: 5! =

RIGHT TRI • r, θ



Paper Step: _____ [ ] SC-32 Ans: _____

2 Tier 1 • Builder

Simplify factorial ratio: 8! ÷ 6! =

RIGHT TRI • r, θ

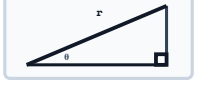


Paper Step: _____ [ ] SC-32 Ans: _____

3 Tier 1 • Builder

Evaluate permutation: 5P2 =

RIGHT TRI • r, θ



Paper Step: _____ [ ] SC-32 Ans: _____

4 Tier 1 • Builder

Evaluate permutation: 6P3 =

RIGHT TRI • r, θ



Paper Step: _____ [ ] SC-32 Ans: _____

5 Tier 1 • Builder

Evaluate combination: 5C2 =

RIGHT TRI • r, θ



Paper Step: _____ [ ] SC-32 Ans: _____

6 Tier 1 • Builder

Evaluate combination: 7C2 =

RIGHT TRI • r, θ



Paper Step: _____ [ ] SC-32 Ans: _____

7 Tier 1 • Builder

Evaluate combination using symmetry: 9C7 =

RIGHT TRI • r, θ



Paper Step: _____ [ ] SC-32 Ans: _____

8 Tier 1 • Builder

Evaluate combination: 8C3 =

RIGHT TRI • r, θ

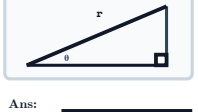


Paper Step: _____ [ ] SC-32 Ans: _____

9 Tier 1 • Builder

Pascal's Row Sum: 4C0 + 4C1 + 4C2 + 4C3 + 4C4 =

RIGHT TRI • r, θ



Paper Step: _____ [ ] SC-32 Ans: _____

10 Tier 1 • Builder

How many distinct rearrangements of the letters in MOM?

RIGHT TRI • r, θ



Paper Step: _____ [ ] SC-32 Ans: _____