Vector Magnitude & Direction: Rectangular to Polar Coordinates

Right Triangle Trigonometry & Vectors • Standard: CCSS.MATH.HSN.VM.A.1

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1. MATHEMATICAL CONCEPT & STACK APPROACH: Decompose 2D vectors into horizontal and vertical components (dx, dy) and calculate polar magnitude r and direction angle ϑ.

Every 2D vector or right triangle relates two perpendicular legs (dx, dy) to a hypotenuse r and angle ϑ via $r = \sqrt{x^2 + y^2}$ and $\tan(\vartheta) = y/x$. Placing vertical component dy in Register Y and horizontal component dx in Register X matches the key legend [→ϑ,r] (placing ϑ in Y and r in X).

2. MATHEMATICAL CONCEPT & STUDENT PREDICTION

Mathematical Focus: Right-Triangle & 2D Vector Component Diagram

Mathematical Setup: Draw a right triangle with vertical rise $dy = 3$ (Register Y) and horizontal run $dx = 4$ (Register X) to model the 3-4-5 vector.

Student Prediction (Before Calculator): Rise = 3, Run = 4. Predict magnitude: $r = \sqrt{4^2 + 3^2} = 5$. Predict angle: arctan(3/4) ≈ 36.87°.

3. CALCULATOR EXECUTION (SC-32 KEYCAPS & 4-LEVEL STACK TRACE)

Keystroke Flow: [3] [ENTER] [4] [↶] [▸ϑ,r] [x⇄y]

Key Pressed	Screen (X)	Internal 4-Level Stack Effect (T, Z, Y, X)
[3]	3	Enter vertical component $y = 3$
[ENTER]	3	$y = 3$ lifted to Waiting Shelf Y
[4]	4	Enter horizontal component $x = 4$ in Workbench X
[↶] [▸ϑ,r]	5.00	Rectangular to Polar! Screen X displays magnitude $r = 5.00$ ($\vartheta = 36.87°$ in Y)
[x⇄y]	36.87	Swap registers: X displays direction angle $\vartheta = 36.87°$!

4. CONTEXT (A Stick in Alexandria: How Eratosthenes Measured the Earth with a Shadow) • Thinking in the Stack (SC-32 Manual) — Ch. 4: Circular Trigonometry & Vector Projections ([→ϑ,r], [→y,x]) + Ch. 5: Complex Numbers & AC Phasors + Ch. 6: Solid Geometry

In 240 BC, Eratosthenes measured a 7.2° shadow angle in Alexandria at noon on the summer solstice while a deep well in Syene 800 km away had zero shadow—calculating Earth's circumference within 1% using pure right-triangle ratios! That same relationship between perpendicular components (dx, dy) and polar angle/distance (r, ϑ) guides modern navigation and engineering.

Core Investigation (Student Warm-Up A):

A surveyor walks 60 m East ($x = 60$) and 80 m North ($y = 80$). Use the Pythagorean theorem and tangent ratio to find the straight-line distance r and angle ϑ north of east.

Teacher Answer Key: $r = \sqrt{60^2 + 80^2} = 100$ m; $\vartheta = \arctan(80/60) \approx 53.13°$.

2D Vector Addition: Resultant Forces by Component Summation

Vector Addition & Resultant Forces • Standard: CCSS.MATH.HSN.VM.A.3

1. FOLLOW-UP EXTENSION CONCEPT, PAPER STEPS & PREDICTION

Concept & Stack Approach: Non-collinear vectors cannot be added by simply summing their lengths; their vertical (y) and horizontal (x) components must be summed separately first. Summing the *y*-components into Y and *x*-components into X prepares $\langle x_{net}, y_{net} \rangle$ directly for [→ϑ,r].

Mathematical Setup & Prediction: Sketch component vectors F1 = ⟨3, 2⟩ and F2 = ⟨1, 5⟩ tip-to-tail on a coordinate grid. Prediction: Add components: $y_{net} = 2 + 5 = 7$; $x_{net} = 3 + 1 = 4$. Magnitude $r = \sqrt{4^2 + 7^2} = \sqrt{65} \approx 8.06$.

2. EXTENSION CALCULATOR SEQUENCE & STACK TRACE

[2] [ENTER] [5] [+] [3] [ENTER] [1] [+] [↶] [▸θ,r]

Key Pressed	Screen (X)	Internal 4-Level Stack Effect
[2]	2	Vertical component y1 = 2
[ENTER]	2	Lifted to Y
[5]	5	Vertical component y2 = 5
[+]	7	Net vertical force $y_{net} = 7$ in X
[3]	3	Horizontal component x1 = 3 (7 lifts to Y)
[ENTER]	3	3 lifted to Y; 7 in Z
[1]	1	Horizontal component x2 = 1 in X
[+]	4	Net horizontal force $x_{net} = 4$ in X; $y_{net} = 7$ waits in Y
[↶] [▸θ,r]	8.06	Resultant force magnitude $r = 8.06$ in X (angle $\vartheta = 60.26°$ in Y)!

3. TARGET MISCONCEPTIONS & VISUAL STACK REMEDIATIONS

Misconception 1: Evaluating trigonometric functions in Radian mode when angles are given in Degrees (or vice versa).

Visual Diagram & Stack Remediation: Have students check the DEG/RAD indicator on the SC-32 status bar and verify on a 30°-60°-90° right-triangle sketch that sin(30°) = 0.5000 before starting vector problems.

Misconception 2: Swapping x and y components when converting rectangular coordinates to polar via [→ϑ,r].

Visual Diagram & Stack Remediation: Label vertical Rise (dy) for Waiting Shelf Y and horizontal Run (dx) for Workbench X on your vector sketch before pressing [→ϑ,r].

Extension Challenge (Student Warm-Up B):

Two cables pull on a post: Vector A is ⟨5, 2⟩ and Vector B is ⟨-2, 4⟩. Find the resultant vector $\langle x_{net}, y_{net} \rangle$ and its magnitude r.

My Paper Answer: _____ **[ ] Verified on SC-32 (Online Key: stackcalc.io/curriculum/answers)**

STUDENT LAB WORKSHEET • UNIT 09 (SC-32)

Trigonometry, Vectors & Polar/Rectangular Coordinates

Name: _____ Date: _____ Class: _____

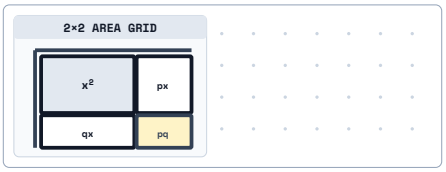
STUDENT REFERENCE GUIDE: RIGHT-TRIANGLE RATIOS & 2D VECTOR DECOMPOSITION RULES

- Trigonometric functions are ratios of right-triangle sides: $\sin(\vartheta)$ = Opposite / Hypotenuse, $\cos(\vartheta)$ = Adjacent / Hypotenuse, $\tan(\vartheta)$ = Opposite / Adjacent.
- Every 2D vector $v = \langle dx,\ dy \rangle$ forms a right triangle with magnitude $r = \sqrt{dx^2 + dy^2}$ and direction angle $\vartheta = \arctan(dy\ /\ dx)$.
- To add two vectors $A = \langle x1,\ y1 \rangle$ and $B = \langle x2,\ y2 \rangle$, add their vertical components (y1 + $y2$) and horizontal components (x1 + $x2$) separately.

Key Sequence: 3 ENTER 4 ↶ ▸θ,r x ⇄ y

Investigation 1: Find the magnitude r and direction angle ϑ for the vector ⟨12, 5⟩.

1. Right Triangle / Vector Sketch:



2. Geometric / Trig Formula Steps:

Formula / Trig Ratio:

Substitute & Square / Eval:

Root / Angle Solution:

3. My Prediction:

_____

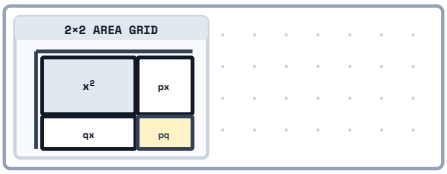
Final Verified Answer:

_____

[ ] Checked on SC-32

Investigation 2: Find the hypotenuse of a right triangle with legs of length 7 and 24.

1. Right Triangle / Vector Sketch:



2. Geometric / Trig Formula Steps:

Formula / Trig Ratio:

Substitute & Square / Eval:

Root / Angle Solution:

3. My Prediction:

_____

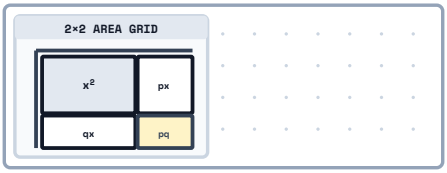
Final Verified Answer:

_____

[ ] Checked on SC-32

Investigation 3: In a 30°-60°-90° right triangle, what is sin(30°)?

1. Right Triangle / Vector Sketch:



2. Geometric / Trig Formula Steps:

Formula / Trig Ratio:

Substitute & Square / Eval:

Root / Angle Solution:

3. My Prediction:

_____

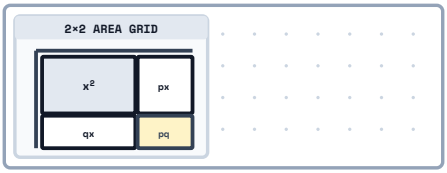
Final Verified Answer:

_____

[ ] Checked on SC-32

Investigation 4: A vector has magnitude $r = 10$ at angle $\vartheta = 45°$. Find its horizontal and vertical components.

1. Right Triangle / Vector Sketch:



2. Geometric / Trig Formula Steps:

Formula / Trig Ratio:

Substitute & Square / Eval:

Root / Angle Solution:

3. My Prediction:

_____

Final Verified Answer:

_____

[ ] Checked on SC-32

UNIT 09 PRACTICE WORKSHEET A • GRADES 9–11 (SC-32)

Unit 09 Practice A: Core Fluency

Name: _____ Date: _____ Score: _____ / 10

TIER 1 • LEVEL 1: BUILDER FOUNDATION • STUDY THESE 2 WORKED EXAMPLES BEFORE SOLVING PRACTICE A (#1–#10) ON PAPER FIRST

EXAMPLE 1 Find the magnitude r of vector ($\Delta x = 3$, $\Delta y = 4$).

$r = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$. Key: [4] [ENTER] [3] [$\rightarrow \theta, r$].

Verified Answer: 5

EXAMPLE 2 Find the distance between (1, 2) and (7, 10).

$\Delta x = 6$, $\Delta y = 8 \rightarrow d = \sqrt{6^2 + 8^2} = \sqrt{100} = 10$.

Verified Answer: 10

1

Tier 1 • Builder

Find the hypotenuse of a right triangle with legs $a = 6$ and $b = 8$:

2×2 AREA GRID



Paper Step: _____ [ ] SC-32 Ans: _____

2

Tier 1 • Builder

Find the hypotenuse of a right triangle with legs $a = 5$ and $b = 12$:

2×2 AREA GRID



Paper Step: _____ [ ] SC-32 Ans: _____

3

Tier 1 • Builder

Find the hypotenuse of a right triangle with legs $a = 8$ and $b = 15$:

2×2 AREA GRID



Paper Step: _____ [ ] SC-32 Ans: _____

4

Tier 1 • Builder

Find the missing leg b if leg $a = 9$ and hypotenuse $c = 15$:

2×2 AREA GRID



Paper Step: _____ [ ] SC-32 Ans: _____

5

Tier 1 • Builder

Find the missing leg a if leg $b = 24$ and hypotenuse $c = 25$:

2×2 AREA GRID



Paper Step: _____ [ ] SC-32 Ans: _____

6

Tier 1 • Builder

Find the distance between (0, 0) and (-9, 12):

2×2 AREA GRID



Paper Step: _____ [ ] SC-32 Ans: _____

7

Tier 1 • Builder

Find the distance between (2, -1) and (5, 3):

2×2 AREA GRID



Paper Step: _____ [ ] SC-32 Ans: _____

8

Tier 1 • Builder

Find the midpoint M of the segment connecting (2, 8) and (10, 4):

2×2 AREA GRID



Paper Step: _____ [ ] SC-32 Ans: _____

9

Tier 1 • Builder

Find the slope $m = (y_2 - y_1)/(x_2 - x_1)$ through (1, 3) and (5, 11):

2×2 AREA GRID



Paper Step: _____ [ ] SC-32 Ans: _____

10

Tier 1 • Builder

Add the 2D vectors $u = \langle 4, -2 \rangle$ and $v = \langle 3, 7 \rangle$:

2×2 AREA GRID



Paper Step: _____ [ ] SC-32 Ans: _____