Factoring $x^2 + 5x + 6$ into $(x + 2)(x + 3)$ with Prime Factor Blocks

Polynomial Equations & Factoring • Standard: CCSS.MATH.HSA.APR.A.1

1. MATHEMATICAL CONCEPT & STACK APPROACH: Factor the constant term with Prime Factor blocks, sketch the (x + p)(x + q) area grid, and find the roots of the quadratic equation.

A polynomial $ax^2 + bx + c$ mirrors base-10 place value with variable powers (x^2, x^1, x^0) in place of 10^2, 10^1, 10^0. Factoring $x^2 + 5x + 6$ into $(x + 2)(x + 3)$ decomposes the constant term into prime factors ($2 \times 3 = 6$) whose sum equals the linear coefficient ($2 + 3 = 5$).

2. MODELING WITH BLOCKS & STUDENT PREDICTION

Block Kit & Mathematical Focus: Prime Factor Studio & Algebraic Area Diagram

Hands-On Setup: Stack Prime Factor blocks [2] and [3] ($2 \times 3 = 6$) in the comparator lid and sketch the 2×2 area grid for $(x + 2)(x + 3) = x^2 + 3x + 2x + 6$.

Student Prediction (Before Calculator): Prime factors of 6 are 2 and 3, and their sum is $2 + 3 = 5$. The factored form is $(x + 2)(x + 3)$ with roots $x = -2$ and $x = -3$.

3. CALCULATOR EXECUTION (SC-32 KEYCAPS & 4-LEVEL STACK TRACE)

Keystroke Flow: [1] [ENTER] [5] [ENTER] [6] [↷] [POLY]

Key Pressed	Screen (X)	Internal 4-Level Stack Effect (T, Z, Y, X)
[1]	1	Coefficient $a = 1$ entered into X
[ENTER]	1	1 lifted to Y
[5]	5	Coefficient $b = 5$ entered into X
[ENTER]	5	b in Y, a in Z
[6]	6	Coefficient $c = 6$ entered into X
[↷] [POLY]	-2	Press [g:] [POLY]: Root 1 = -2 in Y (factor: $x + 2$), Root 2 = -3 in X (factor: $x + 3$)!

4. CONTEXT (Al-Khwarizmi's Geometric Squares & The 1535 Public Algebra Duel in Milan) • Thinking in the Stack (SC-32 Manual) — Ch. 6: Galileo's Square-Cube Area Law + Ch. 7: Polynomial Evaluation, Cardano's Cubic & Brent's Root-Finder ([POLY] & [SOLVE]) + Ch. 8: Romberg Calculus

In 820 AD in Baghdad, Al-Khwarizmi solved $x^2 + 10x = 39$ by drawing literal geometric squares and rectangles ('completing the square')—and in 1535 in Milan, Niccolò Tartaglia defeated Antonio Fiore in a 30-problem public algebra duel overnight! By sketching a 2×2 area grid and factoring constant terms with Prime Factor blocks, students see why (x + p)(x + q) works geometrically.

Core Investigation (Student Warm-Up A):

Factor $x^2 + 7x + 12$ by decomposing 12 with Prime Factor blocks to find the factor pair whose sum is 7, and find both roots of $x^2 + 7x + 12 = 0$.

Teacher Answer Key: $3 \times 4 = 12$ and $3 + 4 = 7$; factored form $(x + 3)(x + 4)$, roots $x = -3$ and $x = -4$.

Signed Factor Pairs: Factoring Trinomials with Negative Terms

Factoring Quadratic Trinomials • Standard: CCSS.MATH.HSA.REI.B.4

1. FOLLOW-UP EXTENSION CONCEPT, BLOCKS & PREDICTION

Concept & Stack Approach: When a quadratic has a negative constant or linear term (such as $x^2 - x - 6$), the factor pair must have opposite signs (-3 and +2) so their product is -6 and their sum is -1. You can check both roots on the calculator after factoring on paper.

Blocks & Prediction: Factor |c| = 6 into Prime Factor blocks [2] and [3] and sketch a 2×2 algebraic area grid for $x^2 - x - 6$. Prediction: Assign signs to the factor pair (2, -3) so their product is -6 and their sum is 2 + (-3) = -1: factors are $(x - 3)(x + 2)$.

2. EXTENSION CALCULATOR SEQUENCE & STACK TRACE

[1] [ENTER] [1] [+/−] [ENTER] [6] [+/−] [↗] [POLY]

Key Pressed	Screen (X)	Internal 4-Level Stack Effect
[1]	1	Coefficient $a = 1$
[ENTER]	1	1 lifted to Y
[1]	1	Enter magnitude of b
[+/−]	-1	Coefficient $b = -1$ in X
[ENTER]	-1	b in Y, a in Z
[6]	6	Enter magnitude of c
[+/−]	-6	Coefficient $c = -6$ in X
[↗] [POLY]	3	Press [g:] [POLY]: Root 1 = 3 in Y (factor: $x - 3$), Root 2 = -2 in X (factor: $x + 2$)!

3. TARGET MISCONCEPTIONS & HANDS-ON BLOCK REMEDIATIONS

Misconception 1: Expanding $(x + 2)(x + 3)$ as $x^2 + 6$ by forgetting the two linear cross-terms ($2x$ and $3x$).

Hands-On Block Remediation: Sketch a 2×2 algebraic area grid for $(x + 2)(x + 3)$ and factor 6 into Prime Factor blocks [2] and [3] so students see that $b = 2 + 3 = 5$ and $c = 2 \times 3 = 6$.

Misconception 2: Using the subtraction operator [−] instead of the change-sign key [+/−] when entering negative polynomial coefficients into [POLY].

Hands-On Block Remediation: Point out that [−] immediately subtracts X from Y, whereas [+/−] negates the number currently in Register X without collapsing the stack.

Extension Challenge (Student Warm-Up B):

Factor $x^2 - 5x + 6$ into two binomials by finding two negative factors of +6 whose sum is -5, and state both roots.

My Paper Answer: _____ [ ] Verified on SC-32 (Online Key: stackcalc.io/curriculum/answers)

STUDENT LAB WORKSHEET • UNIT 08 (SC-32)

Algebraic Expressions, Base-x Polynomials & Quadratic Factoring

Name: _____ Date: _____ Class: _____

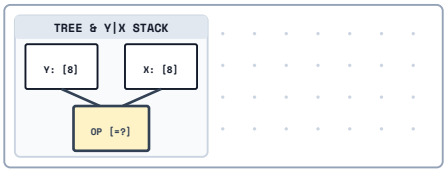
STUDENT REFERENCE GUIDE: QUADRATIC AREA MODEL & PRIME FACTORING RULES

- A polynomial $ax^2 + bx + c$ mirrors base-10 place value with variable powers (x^2, x^1, x^0) in place of 10^2, 10^1, 10^0.
- To factor $x^2 + bx + c$ into $(x + p)(x + q)$, use Prime Factor blocks to factor c into two numbers p and q whose product is c and whose sum is b.
- When c is negative, p and q have opposite signs; when c is positive and b is negative, both p and q are negative.

Key Sequence: `1` `ENTER` `5` `ENTER` `6` `↷` `POLY`

Investigation 1: Factor $x^2 + 6x + 8$ by decomposing 8 into a factor pair that sums to 6, and find the roots.

1. Number Line / Equation Balance:



2. Algebraic Setup & Inverse Steps:

Signed / Equation Setup:

Inverse Operation Step(s):

Verified Solution:

3. My Prediction:

_____

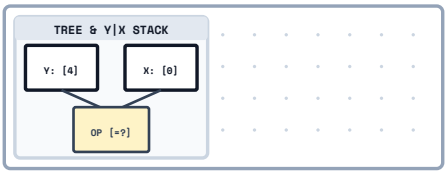
Final Verified Answer:

_____

[ ] Checked on SC-32

Investigation 2: Factor x^2 - 4 (difference of two squares, where the linear coefficient $b = 0$).

1. Number Line / Equation Balance:



2. Algebraic Setup & Inverse Steps:

Signed / Equation Setup:

Inverse Operation Step(s):

Verified Solution:

3. My Prediction:

_____

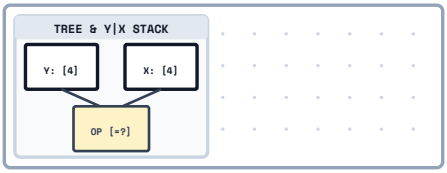
Final Verified Answer:

_____

[ ] Checked on SC-32

Investigation 3: Factor $x^2 + 5x + 4$ into two binomials and state the solutions to $x^2 + 5x + 4 = 0$.

1. Number Line / Equation Balance:



2. Algebraic Setup & Inverse Steps:

Signed / Equation Setup:

Inverse Operation Step(s):

Verified Solution:

3. My Prediction:

_____

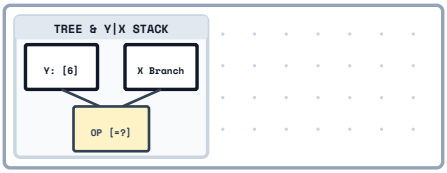
Final Verified Answer:

_____

[ ] Checked on SC-32

Investigation 4: Factor x^2 - $5x + 6$ using signed factor pairs and state both roots.

1. Number Line / Equation Balance:



2. Algebraic Setup & Inverse Steps:

Signed / Equation Setup:

Inverse Operation Step(s):

Verified Solution:

3. My Prediction:

_____

Final Verified Answer:

_____

[ ] Checked on SC-32

UNIT 08 PRACTICE WORKSHEET A • GRADES 7–8 (SC-32)

Unit 08 Practice A: Core Fluency

Name: _____ Date: _____ Score: _____ / 10

TIER 1 • LEVEL 1: BUILDER FOUNDATION • STUDY THESE 2 WORKED EXAMPLES BEFORE SOLVING PRACTICE A (#1–#10) ON PAPER FIRST

EXAMPLE 1 Evaluate $2x^2 - 3x + 5$ when $x = 4$.

$2(16) - 12 + 5 = 32 - 12 + 5 = 25$. Horner: $((2\times4 - 3)\times4) + 5 = 25$.

Verified Answer: 25

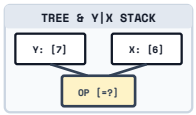
EXAMPLE 2 Simplify by combining like terms & Distributive Property: $3(2x + 4) - x$.

$6x + 12 - x = 5x + 12$.

Verified Answer: $5x + 12$

1 Tier 1 • Builder

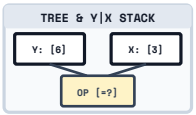
Evaluate $4x + 7$ when $x = 6$:



Paper Step: _____ | | SC-32 Ans: _____

2 Tier 1 • Builder

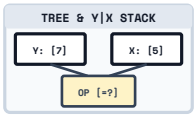
Evaluate $x^2 + 5x + 6$ when $x = 3$:



Paper Step: _____ | | SC-32 Ans: _____

3 Tier 1 • Builder

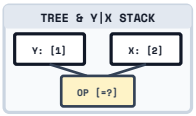
Evaluate $x^2 - 4x + 7$ when $x = 5$:



Paper Step: _____ | | SC-32 Ans: _____

4 Tier 1 • Builder

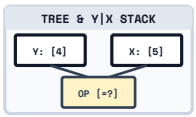
Evaluate $3x^2 - 2x + 1$ when $x = 2$:



Paper Step: _____ | | SC-32 Ans: _____

5 Tier 1 • Builder

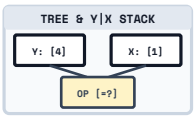
Evaluate $2a + 3b$ when $a = 4$ and $b = 5$:



Paper Step: _____ | | SC-32 Ans: _____

6 Tier 1 • Builder

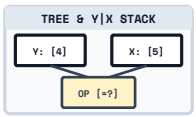
Combine like terms: $7x + 4 + 2x - 1 =$



Paper Step: _____ | | SC-32 Ans: _____

7 Tier 1 • Builder

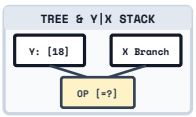
Expand using the Distributive Property: $4(3x - 5) =$



Paper Step: _____ | | SC-32 Ans: _____

8 Tier 1 • Builder

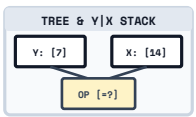
Factor out the GCF: $12x + 18 =$



Paper Step: _____ | | SC-32 Ans: _____

9 Tier 1 • Builder

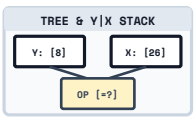
Solve the linear equation: $3x - 7 = 14$



Paper Step: _____ | | SC-32 Ans: _____

10 Tier 1 • Builder

Solve the linear equation: $5x + 8 = 2x + 26$



Paper Step: _____ | | SC-32 Ans: _____