Chained Operations: Evaluating (a + b) × (c - d) from the Inside Out

Numerical & Algebraic Expressions • Standard: CCSS.MATH.6.EE.A.2

1. MATHEMATICAL CONCEPT & STACK APPROACH: Evaluate multi-branch expressions from the inside out using prime factor towers and verify on the 4-register stack without parentheses.

Written algebra uses parentheses to show which inner operations must happen first. Evaluating from the inside out follows true mathematical structure: compute the first branch (6 + 1 = 7), hold that result while computing the second branch (9 - 5 = 4), and multiply the two branch values together.

2. MODELING WITH BLOCKS & STUDENT PREDICTION

Published Block Kit: Prime Factor Studio (Logarithmic Prime Blocks) & 4-Register Stack

Hands-On Setup: Use Prime Factor Studio blocks in the 12-socket comparator lid to model the two factor branches: (6 + 1) = [7] prime block and (9 - 5) = [2] + [2] = 4 tower, then combine their heights for 7 × 4 = 28.

Student Prediction (Before Calculator): Left sub-expression: 6 + 1 = 7. Right sub-expression: 9 - 5 = 4. Combined product: 7 × 4 = 28 (prime tower 2 × 2 × 7).

3. CALCULATOR EXECUTION (SC-6 KEYCAPS & 4-LEVEL STACK TRACE)

Keystroke Flow: [6] [ENTER] [1] [+] [9] [ENTER] [5] [-] [×]

Key Pressed	Screen (X)	Internal 4-Level Stack Effect (T, Z, Y, X)
[6]	6	Value a (6) placed into Register X
[ENTER]	6	ENTER lifts 6 into Register Y
[1]	1	Value b (1) placed into Register X
[+]	7	Left operator (+): Combines Y(6) + X(1) = 7 in X
[9]	9	Value c (9) entered into X; 7 lifted to Y
[ENTER]	9	ENTER lifts 9 into Z and 7 into T
[5]	5	Value d (5) placed into Register X
[-]	4	Right operator (-): Combines Z/ Y(9) - X(5) = 4 in X; 7 drops to Y
[×]	28	Root operator (×): Combines Y(7) × X(4) = 28 in X!

4. CONTEXT (Why 24 Is the Champion Packing Number & How Inside-Out Evaluation Eliminates Parentheses) • From Blocks to the Stack (SC-6 Manual) — Ch. 7: Prime Numbers & Factor Towers + Thinking in the Stack (SC-32 Manual) — Prologue, Ch. 2: Inside-Out Thinking & Ch. 12: Coordinate Bridge

Why are eggs sold by the dozen and hours divided into 24? Because 24 = 2 × 2 × 2 × 3 has eight divisors (1, 2, 3, 4, 6, 8, 12, 24), making it a champion packing number! And when evaluating nested expressions, 16th-century typesetters only invented parentheses to flatten 2D fraction bars onto a single line of type—whereas evaluating from the inside out resolves every branch naturally.

Core Investigation (Student Warm-Up A):

Evaluate (8 + 2) × (14 - 9) by finding the value of each parenthesized group first, then multiplying the two results.

Teacher Answer Key: (8 + 2) = 10 and (14 - 9) = 5; 10 × 5 = 50 (Prime factors: 2 × 5 × 5).

Nested Fractions & Multi-Layer Expressions

Order of Operations & Nested Fractions • Standard: CCSS.MATH.6.EE.A.2

1. FOLLOW-UP EXTENSION CONCEPT, BLOCKS & PREDICTION

Concept & Stack Approach: Deeply nested fractions such as (12 - 4) ÷ (2 × (1 + 1)) require evaluating the numerator (8) and the innermost group of the denominator (1 + 1 = 2, then 2 × 2 = 4) before dividing. The 4-register stack (T, Z, Y, X) mirrors this exact paper-and-pencil hierarchy.

Blocks & Prediction: Model the numerator (12 - 4 = 8, three [2] blocks) and denominator (2 × (1 + 1) = 4, two [2] blocks) with Prime Factor towers. Prediction: Numerator: 12 - 4 = 8 (three [2] blocks). Denominator: 2 × 2 = 4 (two [2] blocks). Result: 8 ÷ 4 = 2.

2. EXTENSION CALCULATOR SEQUENCE & STACK TRACE

[1] [2] [ENTER] [4] [-] [2] [ENTER] [1] [ENTER] [1] [+] [×] [÷]

Key Pressed	Screen (X)	Internal 4-Level Stack Effect
[1]	1	First digit of 12
[2]	12	12 in register X
[ENTER]	12	12 lifted to Y
[4]	4	4 in register X
[-]	8	Numerator 12 - 4 = 8 held in X
[2]	2	Factor 2 enters X; 8 lifts to Y
[ENTER]	2	2 lifted to Y; 8 lifts to Z
[1]	1	Inner 1 enters X
[ENTER]	1	1 lifted to Y; 2 in Z; 8 in T
[1]	1	Second 1 enters X
[+]	2	Innermost sum: 1 + 1 = 2 in X
[×]	4	Denominator combined: 2 × 2 = 4 in X; 8 in Y
[÷]	2	8 / 4 = 2!

3. TARGET MISCONCEPTIONS & HANDS-ON BLOCK REMEDIATIONS

Misconception 1: Evaluating expressions strictly left-to-right (computing 10 + 5 × 2 as 15 × 2 = 30) instead of respecting operator structure.

Hands-On Block Remediation: Model the product branch 5 × 2 with Prime Factor [5] and [2] blocks first: push 10 to Register Y ([10] [ENTER]), evaluate 5 × 2 in X ([5] [ENTER] [2] [×] → 10), then press [+] → 20.

Misconception 2: Losing track of intermediate results when evaluating two parenthesized branches like (8 + 2) × (9 − 4).

Hands-On Block Remediation: Show how Register Y holds the first branch (8 + 2 = 10) automatically while the second branch (9 − 4 = 5) is computed in X, ready for [×] = 50.

Extension Challenge (Student Warm-Up B):

Evaluate (15 - 5) ÷ (3 + 2) by computing the numerator and denominator separately, then dividing.

My Paper Answer: _____ **[ ] Verified on SC-6 (Online Key: stackcalc.io/curriculum/answers)**

STUDENT LAB WORKSHEET • UNIT 07 (SC-6)

Postfix Evaluation, Factor Trees & Chained Operations

Name: _____ Date: _____ Class: _____

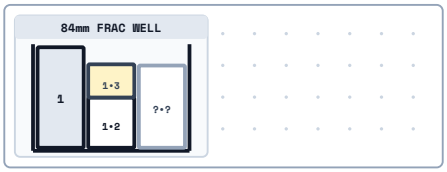
STUDENT REFERENCE GUIDE: INSIDE-OUT EVALUATION & PRIME FACTOR TREE RULES

- Order of operations works from the inside out: evaluate inner groups, parentheses, and factor branches before combining them.
- Every composite number decomposes into a unique tower of prime factors (e.g., $28 = 2 \times 2 \times 7$).
- In nested fractions such as $(a - b) \div (c + d)$, the fraction bar groups the numerator and denominator as separate sub-problems.

Key Sequence: `6` `ENTER` `1` `+` `9` `ENTER` `5` `-` `×`

Investigation 1: Evaluate (4 + 3) × (10 - 6) by solving each parenthesized group first.

1. Factor Tree / Grouping Sketch:



2. Inside-Out Evaluation Steps:

Step 1 (Inner Group / Power):

Step 2 (Next Operation):

Final Value / Prime Factors:

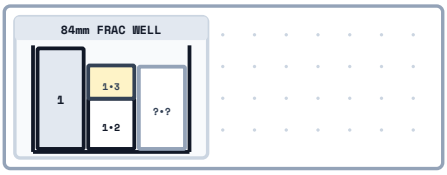
3. My Prediction:

Final Verified Answer:

[ ] Checked on SC-6

Investigation 2: Evaluate (20 - 4) ÷ (2 × 4) by finding the numerator and denominator first.

1. Factor Tree / Grouping Sketch:



2. Inside-Out Evaluation Steps:

Step 1 (Inner Group / Power):

Step 2 (Next Operation):

Final Value / Prime Factors:

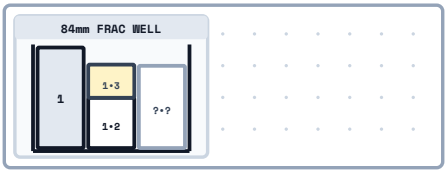
3. My Prediction:

Final Verified Answer:

[ ] Checked on SC-6

Investigation 3: Evaluate (18 - 6) ÷ (1 + 2) and write the prime factorization of the numerator.

1. Factor Tree / Grouping Sketch:



2. Inside-Out Evaluation Steps:

Step 1 (Inner Group / Power):

Step 2 (Next Operation):

Final Value / Prime Factors:

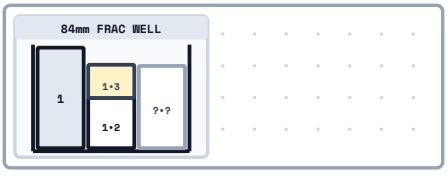
3. My Prediction:

Final Verified Answer:

[ ] Checked on SC-6

Investigation 4: Evaluate 5 + (3 × 4) by computing the inner product before adding 5.

1. Factor Tree / Grouping Sketch:



2. Inside-Out Evaluation Steps:

Step 1 (Inner Group / Power):

Step 2 (Next Operation):

Final Value / Prime Factors:

3. My Prediction:

Final Verified Answer:

[ ] Checked on SC-6

Unit 07 Practice A: Core Fluency

Name: _____ Date: _____ Score: _____ / 10

TIER 1 • LEVEL 1: BUILDER FOUNDATION • STUDY THESE 2 WORKED EXAMPLES BEFORE SOLVING PRACTICE A (#1–#10) ON PAPER FIRST

EXAMPLE 1 **Find the prime factorization of 36.**

$36 = 4 \times 9 = 2^2 \times 3^2$. Key: [36] [FACT].

Verified Answer: $2^2 \times 3^2$

EXAMPLE 2 **Find GCF(18, 24) and LCM(18, 24).**

$18 = 2 \times 3^2$, $24 = 2^3 \times 3$ → Shared GCF $= 2 \times 3 = 6$; Union LCM $= 2^3 \times 3^2 = 72$.

Verified Answer: GCF=6, LCM=72

1 Tier 1 • Builder

Evaluate the power: 2^5 =

84mm FRAC WELL: 1 | 1·3 / 1·2 | ?·?

Paper Step: _____ [ ] SC-6 Ans: _____

2 Tier 1 • Builder

Evaluate the power: 3^4 =

84mm FRAC WELL: 1 | 1·3 / 1·2 | ?·?

Paper Step: _____ [ ] SC-6 Ans: _____

3 Tier 1 • Builder

Write the prime factorization of 24 using exponents:

84mm FRAC WELL: 1 | 1·3 / 1·2 | ?·?

Paper Step: _____ [ ] SC-6 Ans: _____

4 Tier 1 • Builder

Write the prime factorization of 60 using exponents:

84mm FRAC WELL: 1 | 1·3 / 1·2 | ?·?

Paper Step: _____ [ ] SC-6 Ans: _____

5 Tier 1 • Builder

Is 29 a prime number or a composite number?

84mm FRAC WELL: 1 | 1·3 / 1·2 | ?·?

Paper Step: _____ [ ] SC-6 Ans: _____

6 Tier 1 • Builder

Find the Greatest Common Factor: GCF(16, 24) =

84mm FRAC WELL: 1 | 1·3 / 1·2 | ?·?

Paper Step: _____ [ ] SC-6 Ans: _____

7 Tier 1 • Builder

Find the Greatest Common Factor: GCF(30, 45) =

84mm FRAC WELL: 1 | 1·3 / 1·2 | ?·?

Paper Step: _____ [ ] SC-6 Ans: _____

8 Tier 1 • Builder

Find the Least Common Multiple: LCM(6, 8) =

84mm FRAC WELL: 1 | 1·3 / 1·2 | ?·?

Paper Step: _____ [ ] SC-6 Ans: _____

9 Tier 1 • Builder

Find the Least Common Multiple: LCM(12, 15) =

84mm FRAC WELL: 1 | 1·3 / 1·2 | ?·?

Paper Step: _____ [ ] SC-6 Ans: _____

10 Tier 1 • Builder

Use GCF × LCM = $a \times b$: If GCF(12, 18) = 6, find LCM(12, 18):

84mm FRAC WELL: 1 | 1·3 / 1·2 | ?·?

Paper Step: _____ [ ] SC-6 Ans: _____