Mixed Numbers & Height Equivalence: 3/4 + 2 1/2

Fraction Addition & Mixed Numbers • Standard: CCSS.MATH.5.NF.A.1

1. MATHEMATICAL CONCEPT & STACK APPROACH: Represent proper and mixed fractions physically with block towers and computationally with two-dot notation.

Fractions can only be added or subtracted when they share a common unit (denominator), and mixed numbers combine whole units with fractional parts. On the stack, two-dot notation ([.] n [.] d) treats each fraction or mixed number as a single unified value on Y and X.

2. MODELING WITH BLOCKS & STUDENT PREDICTION

Published Block Kit: 84 mm Fraction Towers & Twin Comparator Well

Hands-On Setup: Place three 1/4 blocks (3 × 21 = 63 mm). Beside them, stack two Whole blocks and one 1/2 block (84 + 84 + 42 = 210 mm).

Student Prediction (Before Calculator): Combine heights: 63 mm + 210 mm = 273 mm. In wholes: 3 1/4 (273 / 84 = 3.25).

3. CALCULATOR EXECUTION (SC-6 KEYCAPS & 4-LEVEL STACK TRACE)

Keystroke Flow: [•] [3] [•] [4] [ENTER] [2] [•] [1] [•] [2] [+]

Key Pressed	Screen (X)	Internal 4-Level Stack Effect (T, Z, Y, X)
[•]	0.	Leading dot indicates 0 wholes
[3]	0.3	Numerator = 3
[•]	0.3.	Second dot sets fraction entry mode
[4]	3/4	Denominator = 4: displays 3/4 (or 0.75)
[ENTER]	3/4	3/4 lifted to register Y
[2]	2	Whole = 2
[•]	2.	Dot
[1]	2.1	Numerator = 1
[•]	2.1.	Dot
[2]	2 1/2	Denominator = 2: displays 2 1/2
[+]	3 1/4	Plus combines: 3 1/4 (decimal 3.25)!

4. CONTEXT (The Pharaoh's Bakery & Why the Whole Fraction Tower Stands 84.0 mm Tall) • From Blocks to the Stack (SC-6 Manual) — Ch. 5: Fractions, Decimals & Ratios (The Pharaoh's Bakery & 84mm Comparator)

Four thousand years ago, Egyptian scribes divided 3 loaves of bread fairly among 4 workers by giving each worker 1/2 + 1/4 of a loaf! Because the Whole block in the Fraction Studio is engineered to 84.0 mm (divisible by 1, 2, 3, 4, 6, 7, 8, and 12), a 42 mm (1/2) block plus a 21 mm (1/4) block stacks to 63 mm—matching three 21 mm (3/4) blocks by sight and touch.

Core Investigation (Student Warm-Up A):

Stack a 1/2 tower and a 1/4 tower together. How many 1/4 blocks match their combined height, and what is 1/2 + 1/4?

Teacher Answer Key: 3/4 (three 21 mm fourth-blocks = 63 mm).

Fraction Multiplication & Reciprocals

Fraction Multiplication & Inverses • Standard: CCSS.MATH.5.NF.B.4

1. FOLLOW-UP EXTENSION CONCEPT, BLOCKS & PREDICTION

Concept & Stack Approach: Multiplying (3/4) × (2/3) scales one tower by another—showing unreduced 6/12 first before [SIMP] reduces to 1/2! For fraction division (1/2 ÷ 3/4), use Split & Rebuild (with a 4-kit table pod): split 1/2 (42 mm) into 3 equal 1/6 blocks (14 mm each, ÷3), then stack 4 of them (56 mm = 2/3, ×4) to rebuild 1 full tower!

Blocks & Prediction: Set up 3/4 of a tower (63 mm) and divide into 3 equal parts (21 mm each), taking 2 parts: (3/4) × (2/3). Prediction: 3/4 × 2/3 = 6/12 = 1/2. The resulting block height matches one 1/2 block (42 mm).

2. EXTENSION CALCULATOR SEQUENCE & STACK TRACE

[•] [3] [•] [4] [ENTER] [•] [2] [•] [3] [×]

Key Pressed	Screen (X)	Internal 4-Level Stack Effect
[•]	0.	Start proper fraction 3/4
[3]	0.3	Numerator 3
[•]	0.3.	Fraction separator
[4]	3/4	3/4 in register X
[ENTER]	3/4	3/4 lifted to register Y
[•]	0.	Start proper fraction 2/3
[2]	0.2	Numerator 2
[•]	0.2.	Fraction separator
[3]	2/3	2/3 in register X
[×]	6/12→1/2	Shows 6/12 first; tap [SIMP] to reduce to 1/2 (42 mm)!

3. TARGET MISCONCEPTIONS & HANDS-ON BLOCK REMEDIATIONS

Misconception 1: Adding numerators and denominators directly (e.g., 1/2 + 1/4 = 2/6) or treating 'invert and multiply' for fraction division as an arbitrary rule.

Hands-On Block Remediation: In the comparator lid, 1/2 (42 mm) + 1/4 (21 mm) matches three 1/4 blocks (63 mm), not two 1/6 blocks (28 mm). For division (e.g., 1/2 ÷ 3/4 = 2/3), have a 4-kit table pod combine blocks to Split & Rebuild 1 Full Tower: split 1/2 (42 mm) into three 1/6 blocks (÷3 → 14 mm is 1/4 of the mystery tower), then stack four 1/6 blocks (×4 → 56 mm = two 1/3 blocks) to prove ÷(3/4) = ×(4/3).

Misconception 2: Entering a proper fraction like 3/4 as [3] [.] [4] (which is decimal 3.4) instead of using Two-Dot fraction notation [.][3][.][4].

Hands-On Block Remediation: Teach the rule: 'First dot = whole number zero, second dot = fraction bar.' Typing [.][3][.][4] displays exact 3/4 on the stack.

Extension Challenge (Student Warm-Up B):

Find 2/3 of 3/8 by multiplying (2/3) × (3/8) and simplifying to lowest terms. Which unit fraction tower matches the product?

My Paper Answer: _____ **[ ] Verified on SC-6 (Online Key: stackcalc.io/curriculum/answers)**

STUDENT LAB WORKSHEET • UNIT 06 (SC-6)

Fraction Studio: Height Equivalence & Two-Dot Notation

Name: _____ Date: _____ Class: _____

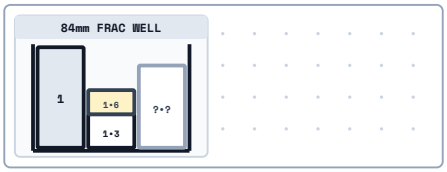
STUDENT REFERENCE GUIDE: TWO-DOT FRACTION ENTRY & TOWER EQUIVALENCE

- One Whole tower stands 84.0 mm tall, dividing cleanly into halves (42 mm), thirds (28 mm), fourths (21 mm), sixths (14 mm), eighths (10.5 mm), and twelfths (7 mm).
- Twin Comparator Well: Stack equivalent fraction towers side-by-side against the 84.0 mm Whole tower to verify equivalence by sight and touch.
- Dividing by a Fraction (Split & Rebuild 1 Full Tower): Just as A ÷ 3 splits tower A into 3 parts to find 1 full tower, A ÷ (c/d) splits tower A into c equal parts (÷ c) and stacks d of them (× d) to build 1 full mystery tower (pooling blocks across a 4-kit table pod when needed).

Key Sequence: [•] [3] [•] [4] [ENTER] [2] [•] [1] [•] [2] [+]

Investigation 1: Add 1/3 + 1/6 and simplify the sum to lowest terms.

1. Fraction Bar / Ratio Sketch:



2. Equivalent Fraction / Decimal Work:

Common Unit / Denominator:

Operation Step:

Simplified Answer:

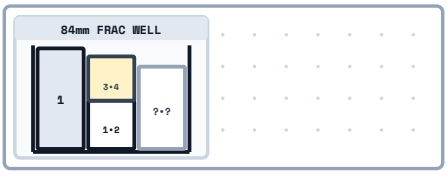
3. My Prediction:

Final Verified Answer:

[ ] Checked on SC-6

Investigation 2: Subtract 1 1/2 - 3/4. What fraction remains?

1. Fraction Bar / Ratio Sketch:



2. Equivalent Fraction / Decimal Work:

Common Unit / Denominator:

Operation Step:

Simplified Answer:

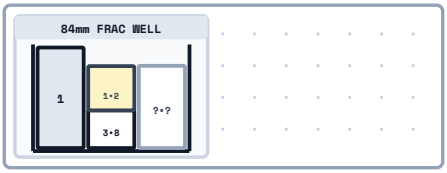
3. My Prediction:

Final Verified Answer:

[ ] Checked on SC-6

Investigation 3: Compare 3/8 to 1/2. Which is smaller, and what is 3/8 written as a decimal?

1. Fraction Bar / Ratio Sketch:



2. Equivalent Fraction / Decimal Work:

Common Unit / Denominator:

Operation Step:

Simplified Answer:

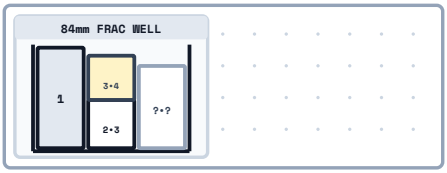
3. My Prediction:

Final Verified Answer:

[ ] Checked on SC-6

Investigation 4: Find the area of a rectangle measuring 2/3 unit by 3/4 unit by multiplying (2/3) × (3/4) and simplifying with [SIMP].

1. Fraction Bar / Ratio Sketch:



2. Equivalent Fraction / Decimal Work:

Common Unit / Denominator:

Operation Step:

Simplified Answer:

3. My Prediction:

Final Verified Answer:

[ ] Checked on SC-6

UNIT 06 PRACTICE WORKSHEET A • GRADES 4–5 (SC-6)

Unit 06 Practice A: Core Fluency

Name: _____ Date: _____ Score: _____ / 10

TIER 1 • LEVEL 1: BUILDER FOUNDATION • STUDY THESE 2 WORKED EXAMPLES BEFORE SOLVING PRACTICE A (#1–#10) ON PAPER FIRST

EXAMPLE 1 How many 1/6 towers match one 1/2 tower (42 mm)?

1/2 = 3/6 because 3 × 14 mm = 42 mm.

Verified Answer: 3/6

EXAMPLE 2 Simplify 8/12 to lowest terms.

Divide numerator and denominator by GCF(8, 12) = 4 → 2/3.

Verified Answer: 2/3

1 Tier 1 • Builder

Find the missing numerator: 1/2 = [?]/8



Paper Step: _____ [ ] SC-6 Ans: _____

2 Tier 1 • Builder

Find the missing numerator: 2/3 = [?]/12



Paper Step: _____ [ ] SC-6 Ans: _____

3 Tier 1 • Builder

Find the missing numerator: 3/4 = [?]/12



Paper Step: _____ [ ] SC-6 Ans: _____

4 Tier 1 • Builder

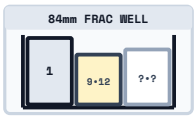
Simplify to lowest terms: 6/8 =



Paper Step: _____ [ ] SC-6 Ans: _____

5 Tier 1 • Builder

Simplify to lowest terms: 9/12 =



Paper Step: _____ [ ] SC-6 Ans: _____

6 Tier 1 • Builder

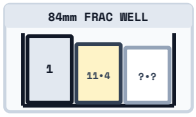
Simplify to lowest terms: 10/15 =



Paper Step: _____ [ ] SC-6 Ans: _____

7 Tier 1 • Builder

Convert improper fraction to mixed number: 11/4 =



Paper Step: _____ [ ] SC-6 Ans: _____

8 Tier 1 • Builder

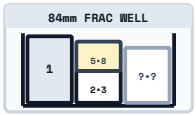
Convert mixed number to improper fraction: 3 2/5 =



Paper Step: _____ [ ] SC-6 Ans: _____

9 Tier 1 • Builder

Compare using <, >, or =: 2/3 [?] 5/8



Paper Step: _____ [ ] SC-6 Ans: _____

10 Tier 1 • Builder

Convert fraction 3/4 to a decimal and a percent:



Paper Step: _____ [ ] SC-6 Ans: _____